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Gerbes and Massive Type II Configurations

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Abstract: We find novel bound states of NS5, D6 and D8-branes in massive type IIA string theory. As the NS gauge transformations can change the Chern class of the RR field these configurations should be thought of as nonlocal objects called gerbes. We develop a global formalism for theories that involve massive tensor fields in general, and apply it in massive type IIA supergravity. We investigate the T-dual NS5/D5-brane configurations in type IIB, and relate them to an F-theory compactification on Calabi–Yau three-fold. We comment on the implications to consistency conditions for brane-wrapping and the classification of D-brane charges in terms of K-theory classes.

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Contents

1. Introduction

Gauge symmetry has certainly proved to be one of the most efficient tools both in quantum field theory and in string theory. Physically it facilitates the study of complicated systems by enlarging the space where these systems are embedded, and ensures the stability of, for instance, such solitonic objects as instantons. These physical ideas are usually formulated mathematically in the language of fiber bundles, which are global constructions that encode much relatively easily accessible information of the topology of the underlying manifold. In this article we give examples of string theory configurations that go beyond the realm of globally defined bundles and belong in a sense to a stringy extension of gauge theories.

In a well defined theory two symmetries might seem to conflict in that one of them could shift the conserved charge of the other. This situation arises for instance in any theory that involves massive tensor fields, such as many world-volume effective theories in string theory, or the massive type IIA supergravity. There a gauge symmetry of one field seems to shift the characteristic class of another field. This situation seems to fall outside the usual mathematical discussion that makes use of everywhere defined bundles. Recently, there have appeared constructions that seem to be able to accommodate this kind of situations [1, 2]. These constructions, *gerbes*, are most conveniently described in terms of *locally* defined line bundles, as opposed to the everywhere defined fiber bundles we referred to above. They are the next logical step in a sequence that starts with functions on a manifold, and is followed by line bundles. These constructions have already been applied in quantum field theory in [3, 4, 5].

In this paper we present some inherently nonlocal objects in string theory, and show how to analyze them as gerbes. These objects arise naturally when one employs the symmetries of the effective type IIA supergravity theory to the full. The example at hand is a bound state of NS5-branes and D6-branes in the presence of a cosmological constant. Gerbes cannot be seen as local objects in the same sense as normal bundles or Chan–Paton bundles can. This fact also sheds light on some of the difficulties encountered in classifying type IIA D-branes in terms of fiber bundles.

In [6] it was suggested that D-brane charges could be described by K-theory classes. The classification of a large class of the cases encountered in string theory was performed (for trivial NS two-form backgrounds) in [7], and continued in [8, 9, 10]. The classification was based on the idea that supersymmetric D-branes could be built as bound states of non-supersymmetric higher-dimensional D-branes [11]. The stability of a these brane–anti-brane constructions depends on the properties of the branes' normal and Chan–Paton bundles. On the other hand, vector bundles on a manifold X are in general classified in K-theory. This leads to a partial classification of D-brane charges in terms of the groups $K(X)$ in type IIB and $K^1(X)$ in type IIA. The classification was also extended to comprise the so-called twisted fiber bundles, which are essentially trivial gerbes. Having introduced gerbes as generalizations of fiber bundles the next logical step would be to extend this classification to something that should then be the "K-theory of gerbes", in order to cope with general NS two-form backgrounds and the cosmological constant in the case of type IIA. This still remains to be done.

JHEP07(1999)002

In all of these configurations the integral cohomology class of the NS field strength plays a central role. It appears as an obstruction to wrapping a brane on a submanifold or, in certain cases, to classifying D-brane charges in K-theory, *and* it is the characteristic class of a gerbe. These appearances are intimately connected. In this paper we attempt among other things to clarify the role played by the two-form NS field in string theory. In the same way that a line bundle has a natural characteristic class related to, say, in the case of the Dirac monopole bundle to the number of monopoles, the characteristic class of the gerbe will in our constructions describe the number of NS5-branes. The local line bundles needed for defining a gerbe imply that there are, in a certain sense, nonlocal D6-branes. Nevertheless, we can find a globally defined modified eight-form field strength. The topological stability of these constructions follows from the gerbe structure.

Another central ingredient will be the zero-form field strength in type IIA string theory, the cosmological constant. We shall consider it a section of a constant sheaf; Allowing it to jump would indicate the presence of a D8-brane. For the most part of the paper the cosmological constant will be a true, though non-zero, constant as turns out to be necessary for the consistency of a gerbe. String theory would seem to resolve, as we shall argue, at least those problems that arise from the presence of the cosmological constant by simply relating the problematic type IIA solutions to more classifiable ones in type IIB.

In order to gain further understanding of these systems we find the T-dual solutions on the type IIB side. These turn out to include a non-trivial axion, NS5/D5-brane dyons, D7-branes and orbifold singularities. Stability considerations in IIA lead one to a consistent treatment of the deficit angle in type IIB. The form-field equations of motion can be solved explicitly, but the metric and the dilaton must be extracted from an F-theory construction on an elliptically fibered Calabi–Yau three-fold.

The article is organized as follows: We begin with a toy model that involves massive tensor fields in general. We then give a brief account of what a gerbe is. We present our ansatz in massive type IIA and solve the form-field equations of motion, dualize the solutions, and relate the construction to an F-theory solution. We then discuss the classification of in particular type IIA branes in K-theory and the consistency conditions for brane wrapping in the last section.

2. Global structures in field theory

2.1 Massive form-fields

The constructions that we shall make use of below are characteristic to any theory, where tensor fields become massive. These theories include the Maxwell–Proca theory, Romans' massive IIA supergravity [12], and effective world-volume theories on branes [13, 14]. In order to extract the essential features of these systems, it may be helpful to start by considering the toy model

$$S = \int_X \mathrm{d}C \wedge *\mathrm{d}C + (mC + \mathrm{d}L) \wedge *(mC + \mathrm{d}L) + \cdots . \tag{2.1}$$

Here C is a $(p+1)$-form field over a space-time X, L is a p-form sometimes called the Stückelberg field, and m is a constant. The action is invariant under the transformations

$$\begin{aligned} C &\longrightarrow C + \mathrm{d}A \\ L &\longrightarrow L - mA \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} C &\longrightarrow C \\ L &\longrightarrow L + \mathrm{d}a\,. \end{aligned} \tag{2.3}$$

In general these transformations remain symmetries of the theory, when e.g. $\mathrm{d}A$ is replaced by a closed form, say F. In this case, however, A is defined only locally.

In order to see what physical states there are one has to fix the gauge. Any configuration (C, L) lies locally on the gauge orbit of $(C + \mathrm{d}L, 0)$. The L-field is thus not physical, and can be gauged away. In this gauge C is obviously a massive tensor field, the mass being given by the parameter m. One may ask what happens in the theory when one takes the limit $m \to 0$. The two fields decouple, and the physics is described by a massless tensor field and a U(1) gauge theory. The physical Hilbert spaces of the theory are rather different depending on the value of m.

2.2 Transition functions

Physical states can generally be described as equivalence classes of fields modulo symmetries. When we are building a theory on a topologically non-trivial space we must first cover the space (say, X) by a suitable collection of local charts $\{\mathcal{U}_\alpha\}$. Fields defined on different charts are related by symmetries of the theory — this means simply that the actions of transition functions on different fields are given essentially by gauge transformations. In this well known way e.g. tensor fields become sections of $\Omega^*(X, \mathbb{R})$.

A particular configuration is described so that one gives the (not yet gauge fixed) fields on each coordinate patch, and tells what gauge transformation $g_{\alpha\beta}$ has to be done on the intersection $\mathcal{U}_\alpha \cap \mathcal{U}_\beta$ to interpolate between different local descriptions. For instance, in the example above we can take $C \in \Omega^{p+1}(X, \mathbb{R})$ and set

$$C_\beta = g_{\beta\alpha} \cdot C_\alpha = C_\alpha + \mathrm{d}A_{\alpha\beta}\,. \tag{2.4}$$

From this construction certain consistency conditions ensue. For example the transition functions of a vector bundle must obey the cocycle condition

$$(\delta g) = g_{\alpha\beta} g_{\beta\gamma} g_{\gamma\alpha} = 1 \tag{2.5}$$

on triple intersections of local charts. Gluing the local theories together on a given space is not always possible; obstructions to this are most conveniently discussed in Čech cohomology, cf. section 3.1.

Let us apply this to the toy model of section 2.1: Given an atlas $\mathcal{U}_\alpha$ on the space X we can define a field theory where fields on different patches are related by the gauge transformations (2.2) and (2.3). The latter transformations suggest for $p = 1$ an underlying line bundle $\mathcal{L} \to X$, provided the global extension of the local form $\mathrm{d}L$ defines an integral cohomology class. This class characterizes the topology of the bundle. The former gauge transformations have a similar action on C. These transformations, however, have a peculiar action on the line bundle $\mathcal{L}$; it seems as if we could shift the characteristic class of the bundle $\mathcal{L}$ *by a symmetry of the theory*. This suggests to consider structures where there are different line bundles on different coordinate patches. These structures are most conveniently described in terms of gerbes.

3. Gerbes

As was in the case of the "ambiguities" introduced by gauge symmetries, even the ambiguity in defining the right line bundles for Stückelberg fields can be turned into an asset. In the same way that the former lead us to work with fiber bundles, the latter should lead us to consider gerbes. Here we shall follow the lecture notes of Hitchin [2].

3.1 Čech cohomology

Let $\mathbb{F} = \mathbb{Z}_2$ or $\mathrm{U}(1)$ and assign to each n-fold intersection of local charts a map to $\mathbb{F}$

$$f_{i_1,\cdots,i_n} : U_{i_1} \cap \cdots \cap U_{i_n} \longrightarrow \mathbb{F}\,. \tag{3.1}$$

that is antisymmetric in its indices. Then we can define a coboundary operator by setting

$$(\delta f)_{i_1,\cdots,i_{n+1}} = \exp \sum_{k=1}^{n+1} (-1)^{k+1} \log f_{i_1,\cdots,\hat{k},\cdots,i_{n+1}}\,. \tag{3.2}$$

For example, the transition functions of an orthogonal or a unitary vector bundle define such functions for $n = 2$, namely $f_{\alpha\beta} = \det(g_{\alpha\beta})$.

The cohomology $\check{H}^*(X, \mathbb{F})$ of this complex, namely the cocycles (i.e. functions that satisfy $\delta f = 0$) modulo coboundaries (i.e. exact functions $f = \delta h$), is called Čech cohomology. It characterizes the topology of the manifold and not only the choice of cover, if the cover is fine enough [15]. For instance, the first Stiefel–Whitney class $w_1 \in \check{H}^1(X, \mathbb{Z}_2)$ which is constructed using transition functions of a tangent bundle is an obstruction to orientability. The second Stiefel–Whitney class w_2 similarly over $\mathbb{Z}_2$ is constructed using lifts of these transition functions into the spin-cover. A non-trivial w_2 obstructs spin-structure.

3.2 Definitions

A space X together with a collection of maps $g_{\alpha\beta\gamma}$ as in (3.1) for $n = 3$ is called a gerbe [2] if

$$(\delta g)_{\alpha\beta\gamma\delta} = 1\,. \tag{3.3}$$

This is an obvious analogue of the condition (2.5). In the same way that a Chern class characterizes a line bundle, a class in $\check{H}^3(X, \mathbb{Z})$ characterizes a gerbe.

There is an equivalent but a somewhat more concrete definition of a gerbe that makes use of local line bundles [2]. It postulates that there exist

1. A line bundle $\mathcal{L}_{\alpha\beta}$ on each intersection $\mathcal{U}_\alpha \cap \mathcal{U}_\beta$.
2. An isomorphism $\mathcal{L}_{\alpha\beta} \simeq \mathcal{L}^{-1}_{\beta\alpha}$.
3. A trivialization $\theta_{\alpha\beta\gamma}$ of

$$\mathcal{L}_{\alpha\beta} \otimes \mathcal{L}_{\beta\gamma} \otimes \mathcal{L}_{\gamma\alpha} \tag{3.4}$$

   that satisfies $(\delta\theta)_{\alpha\beta\gamma\delta} = 1$.

The local trivialization of a gerbe on a two-fold intersection of local charts $\mathcal{L}_{\alpha\beta}$ is a *difference of line bundles*,

$$\mathcal{L}_{\alpha\beta} = \mathcal{L}_\alpha \otimes \mathcal{L}_\beta^{-1}\,. \tag{3.5}$$

This should be seen as an analogue of the observation that a difference (quotient) of two trivializations of a line bundle (i.e. non-zero sections) is an ordinary function. In general it is possible to define higher gerbes as differences of lower gerbes; here we shall only make use of the one-gerbes defined above.

3.3 Connection

Similarly to the curvature two-form of a line bundle that can be used to represent the first Chern class, there is a three-form that can represent the characteristic class of a gerbe. The three-form is constructed using a locally defined two-form gauge potential $B_\alpha \in \Omega^2(\mathcal{U}_\alpha)$. The forms must satisfy the compatibility condition that the difference on a two-fold intersection $\mathcal{U}_\alpha \cap \mathcal{U}_\beta$

$$B_\alpha - B_\beta = F_{\alpha\beta} \tag{3.6}$$

be a curvature of a *local line bundle*. The curvature of the gerbe is then simply a global three-form $[G] = \mathrm{d}B_\alpha \in H^3(X, \mathbb{R})$. Similarly to the Chern class of a line bundle, also the curvature of a gerbe must be in integral cohomology.

This is exactly the situation we encountered in the massive tensor theories: a local two-form that can shift the characteristic class of a local line bundle by a symmetry of the theory.

4. An ansatz in massive type IIA

Type IIA string theory provides a physical example of a system, where the considerations of the previous section are realized. There the mass parameter on the level of supergravity [12] arises as the vacuum expectation value of the dual of a top-form field strength, cf. [16, 17]. Furthermore, *both* of the equivalents of the fields C and L turn out to carry charges associated to nonperturbative excitations in the full string theory, namely NS5-branes and D6-branes. It is therefore necessary to understand the spectrum in the $m \neq 0$ phase without going directly to the naive unitary gauge $L = 0$.

Below we shall present an ansatz, and show that it is consistent with all Bianchi identities and form-field equations of motion. In section 6.2 we relate the T-dual of the ansatz to a system of fivebrane dyons in F-theory compactified on a Calabi–Yau three-fold.

4.1 The modified field strength

Let C be the free sum of RR fields. The modified field strengths of type II field theories are [18]:

$$R(C) = \mathrm{d}C - G \wedge C + m\mathrm{e}^B \,, \tag{4.1}$$

where B is the NS two-form, $G = \mathrm{d}B$, and m the mass parameter of the type IIA theory. The Bianchi identities are

$$\mathrm{d}R(C) - G \wedge R(C) = 0 \,. \tag{4.2}$$

The equations of motion follow from Romans' action [12], which in our notation is

$$\begin{aligned} S = \int \mathrm{e}^{-2\phi} \left(*R + 4\mathrm{d}\phi \wedge *\mathrm{d}\phi - \frac{1}{2} G \wedge *G \right) - \\ - \frac{1}{2} R(C) \wedge *R(C) + \frac{1}{2} \mathrm{d}^{-1}(R^{[4]} \wedge R^{[4]} \wedge G) \end{aligned} \tag{4.3}$$

the only non-trivial components of $R^{[2k]}$ being those for $2k = 0, 2, 4$. Dualizing in the path integral leads to the dual field strengths $R^{[10-2k]} = (-)^k * R^{[2k]}$ as in [19].

Let $A_{\rm CP}$ be the Chan–Paton gauge potential supported on the brane world-volumes. Then the RR form symmetry acts on the various fields according to [18]

$$\begin{aligned} \delta_{\rm RR} C &= {\rm d}\Lambda - G \wedge \Lambda + \lambda {\rm e}^B \\ \delta_{\rm RR} B &= 0 \\ \delta_{\rm RR} A_{\rm CP} &= 0\,. \end{aligned} \tag{4.4}$$

The system possesses also a NS symmetry, namely

$$\begin{aligned} \delta_{\rm NS} C &= -m\eta\, {\rm e}^B \\ \delta_{\rm NS} B &= {\rm d}\eta \\ \delta_{\rm NS} A_{\rm CP} &= 2\pi\eta\,. \end{aligned} \tag{4.5}$$

As explained above, when we build a twisted theory on a manifold it does not matter what the exact values of C and B are. Instead, the physical condition is that the modified field strengths $R(C)$ and G be globally defined sections of $\Omega^*(X)$. The RR symmetry is the ordinary form symmetry in disguise. We can, again, allow ${\rm d}\eta \longrightarrow f$ to be cohomologously non-trivial. Then the NS symmetry, for $m \neq 0$, changes the Chern classes or intersection numbers of the RR fields.

4.2 Topology and RR fields

We consider an ansatz that supports NS5-brane, D6-brane and possibly D8-brane charges in massive type IIA theory. These configurations including their supersymmetries have been discussed in a different context for example in [20, 21]. Many non-supersymmetric solutions of the massive type IIA supergravity were constructed already in Ref. [12]. Supersymmetric solutions include the D8-brane of [22] and [17]. Other solutions were found in [19, 23].

In order to find the relevant perturbative Lagrangian we first consider the Bianchi identities (4.2). It will turn out to be consistent to set $C^{[3]} = C^{[5]} = 0$ and to assume $B \wedge B = 0$. From the duality equations it then follows that

$$\begin{aligned} {\rm d}C^{[7]} &= -*({\rm d}C^{[1]} + mB) & (4.6) \\ {\rm d}C^{[9]} - G \wedge C^{[7]} &= *m\,. & (4.7) \end{aligned}$$

From (4.6), (4.7) and the fact that we shall be able to choose a B-field that satisfies $B \wedge B = 0$ it follows that it is sufficient to consider the action

$$\begin{aligned} S_0 = \int {\rm e}^{-2\phi} \left(*R + 4{\rm d}\phi \wedge *{\rm d}\phi - \frac{1}{2} G \wedge *G \right) - \\ - \frac{1}{2} R^{[8]} \wedge *R^{[8]} + \int C^{[7]} \wedge \tau\,, \end{aligned} \tag{4.8}$$

where τ is the Poincaré dual of the D6-brane. Also the anomalous couplings should be included, but it turns out that the equations of motion remain the same provided ${\rm Tr}(F \wedge F \wedge F) = 0$, where $F = {\rm d}A_{\rm CP}$.

Consider[1] a space-time locally of the form $M^{1,5} \times \mathbb{R} \times S^3$, and a fixed point $p \in S^3$. We cover the sphere with two hemispheres $\mathcal{U}_0$ and $\mathcal{U}_1$, so that the point p is outside $\mathcal{U}_0$. Let H satisfy

$$\Delta H = [V] - 2\pi\delta_p \in H^3(S^3, \mathbb{Z})\,, \tag{4.9}$$

where $[V]$ is the de Rham class of the volume form i.e. generator of $H^3(S^3, \mathbb{Z})$, δ_p is the Poincaré dual of the point p, Q is a parameter, and $\Delta = \{\mathrm{d}, \mathrm{d}^\dagger\}$ is the Hodge-de Rham Laplacian[2]. On the coordinate patch $\mathcal{U}_1$ we can similarly define

$$\Delta H_1 = [V]\,. \tag{4.10}$$

For finding the three-forms H, H_1 it was necessary that the relevant cohomology classes on the right-hand side vanish.

We can now give our ansatz for the various form-fields. The NS two-form is locally defined

$$B = \begin{cases} Q\ \mathrm{d}^\dagger H & \text{on } \mathcal{U}_0 \\ Q\ \mathrm{d}^\dagger H_1 & \text{on } \mathcal{U}_1\,. \end{cases} \tag{4.11}$$

The field strength

$$G = \mathrm{d}B = Q\ [V] \tag{4.12}$$

is globally defined. This ansatz supports Q NS5-branes that are cut out of the origin in $\mathbb{R}^4 \backslash \{0\} \simeq \mathbb{R} \times S^3$.

For generality we consider the mass parameter in Romans' supergravity as a locally constant $\mathbb{R}$-sheaf

$$m = \begin{cases} m_0 & \text{on } \mathcal{U}_0 \\ m_1 & \text{on } \mathcal{U}_1\,. \end{cases} \tag{4.13}$$

This may lead to D8-brane charge. Before introducing the metric the D8-brane is not exactly localized. On the intersection of charts $\mathcal{U}_0 \cap \mathcal{U}_1$ the charge is the difference in cosmological constants $q(D8) = m_1 - m_0$.

As we are interested in bound states that involve D6-branes the correct electric RR-form is $C^{[7]}$, whereas $C^{[1]}$ should be regarded as its magnetic dual. These are naturally both gauge dependent objects, and the field that we should match over the intersections is $R^{[8]}$. It is convenient to consider the difference

$$f = \left(\mathrm{d}C^{[1]}\right)_1 - \left(\mathrm{d}C^{[1]}\right)_0 = m_0 B_0 - m_1 B_1 \tag{4.14}$$

[1]The present ansatz makes use of a construction in [2].

[2]The Hodge star $*$ and the operator $\mathrm{d}^\dagger$ refer to operations on S^3 and to the metric normalized to $\int *1 = 2\pi$.

and to assume e.g. $(\mathrm{d}C^{[1]})_0 = 0$. The D6-brane charge can be calculated integrating over the charge density on a disc $D \subset \mathcal{U}_1$

$$\int_D \mathrm{d}f = \int_D (m_0 - m_1)\ Q\ V - 2\pi m_0 Q\ \delta_p \tag{4.15}$$

$$= (m_0 - m_1)\ Q\ \mathrm{vol}(S^2)\ -\ 2\pi m_0 Q\,. \tag{4.16}$$

Let us now investigate the various field strengths in detail: considering $m_0 = m_1 = m$ we only see the contribution coming from the extracted point. The physical interpretation is that in that point a Dirac string emerges through the sphere $S^3 \subset \mathbb{R}^4$: this is just the D6-brane.

In order to see what happens if $m_0 \neq m_1$ we should modify the above calculation by setting $\Delta H = [V]$, which is possible locally on $\mathcal{U}_0$. The total NS field strength is globally defined; physically this means that we can with impunity put NS5-branes inside a D8-brane. We can stretch the disc D over which integration is performed as to cover all of the sphere but a point. In this limit the integral yields

$$\int_{\mathbb{R}^3} \mathrm{d}f = 2\pi\ (m_0 - m_1)\ Q\,. \tag{4.17}$$

This means that there is a flux corresponding to $(m_0 - m_1)Q$ D6-branes at the excluded point. A similar phenomenon was observed in [21], where passing a NS5-brane through a D8-brane produced a D6-brane tube between the NS5 and D8-branes.

The two-fold intersection of charts $\mathcal{U}_0 \cap \mathcal{U}_1$ is topologically a cylinder. We shall find it convenient to describe it as a line bundle $\pi : C \longrightarrow S^2$ over a two-sphere. As f is closed outside p we can write for concreteness

$$f = \frac{mQ}{2}\ \mathrm{d}\cos\theta \wedge \mathrm{d}\varphi\,, \tag{4.18}$$

where (r, θ, φ) are the local coordinates and we are sufficiently near p. As the formula does not depend on r it descends naturally to a two-form on the sphere S^2. Further away from p the formula depends on the details of the fibration of C. If we can show that f is in integral cohomology we have constructed a gerbe. We defer this discussion to section 4.3. The flux f is not defined far a way from p, but it is purely a property of the cylinder intersection of the two charts. The gauge invariant physical quantity is $R^{[8]} = - * R^{[2]}$. On $\mathcal{U}_0$ we have $R^{[2]} = mB_0$. This actually extends to the whole sphere as $R^{[2]} = mQ\mathrm{d}^\dagger H$. The electric field strength is

$$R^{[8]} = \frac{mQ}{b}\ \mathrm{d}h \wedge \mathrm{Vol}(M^{1,5} \times \mathbb{R}), \tag{4.19}$$

where b is the conformal factor in front of the sphere metric in ten dimensions. The function h satisfies

$$\nabla^2 h = 1 - 2\pi\delta^{(3)}(p)\,. \tag{4.20}$$

The equation of motion is satisfied provided we insert the following six-brane source

$$\tau = mQ\ (V - 2\pi\ \delta_p)\,. \tag{4.21}$$

The D6-branes are supported in p *and* spread evenly over the sphere. The NS two-form equation of motion reduces to ensuring that

$$\mathrm{e}^{2\phi}\ \frac{\mathrm{vol}(M^{1,5}\times\mathbb{R})}{\mathrm{vol}(S^3)} \tag{4.22}$$

does not depend on the coordinates of the sphere.

4.3 Charge quantization

We want to interpret the non-gauge invariant two-form f as the curvature of a suitable line bundle on the cylinder $C : \mathcal{U}_0 \cap \mathcal{U}_1 \simeq S^2 \times \mathbb{R}$. If this can be done the whole of the structure extends to a gerbe, as was explained in section 3. The bundle we are looking for $\mathcal{L}_{01} \longrightarrow S^2$ can be taken to have the same base space S^2 as the cylinder itself. The condition for $\mathcal{L}_{01}$ to be a globally on S^2 defined line bundle is that the curvature two-form be in integral cohomology

$$\left[\frac{f}{2\pi}\right] \in H^2(S^2,\mathbb{Z})\,. \tag{4.23}$$

As the factor $\mathrm{vol}(S^2)$ in the formula for the flux (4.16) depends continuously on the choice of D^3, we are lead to the quantization conditions

$$m_0 = m_1 \tag{4.24}$$

$$m_0 Q \in \mathbb{Z}\,. \tag{4.25}$$

The first one implies that a D8-brane would separate gerbes from each other. The cosmological constant m_0 is quantized [18] because of the Chern–Simons terms in the anomalous couplings. Here this leads to a quantization of Q as well.

Without the help of gerbes it would be difficult to see how the D6-brane charge should be quantized, or conserved, as the field strength f is not globally defined, and as its Chern class can be changed by NS gauge transformations. Here the quantization follows from imposing the gerbe structure explained in section 3. Conservation follows from the fact that the flux arises from comparing two B-fields; even if the B-fields are shifted by a non-trivial two-form in the cohomology of the three-sphere (minus the point) the flux remains the same, provided $m_1 = m_0$. This is exactly the condition that we stay inside a given gerbe.

The quantity that is invariant in field theory is $R^{[2]}$. In our example there are no electric sources for this field $\mathrm{d} * R^{[2]} = 0$, but

$$\mathrm{d}R^{[2]} = m_0 Q\ (V - 2\pi\delta_p)\,. \tag{4.26}$$

This integrates to zero over the sphere, which just means that the sphere is able to absorb the flux emanating from p. From the field theory point of view this is a consequence of the quadratic coupling between B and $\mathrm{d}C^{[1]}$ that turns flux into the B-field [13, 14]. The closest parallel to this is the anomalous coupling between the Chan–Paton field strength and the RR potentials. There will be a net flux out of the sphere only if it is placed on top of the D8-brane, where the flux can flow.

The flux of G emanates from the origin where there is a source. Except exactly at the D6-brane this flux turns into the *charge density* of $R^{[2]}$ implying a monopole distribution spread evenly over the sphere. Of course once we embed the sphere in $\mathbb{R}^4$ the density falls off with the distance from the NS5-branes. The physical sources in string theory are the D6-branes with which one associates the locally defined beam of flux f through the point p.

5. T-duals in type IIB

Massive type IIA solutions can be related to type IIB solutions using a Scherk–Schwarz compactification, where the compact coordinate $z \in S^1$ has period l_B. Applying T-duality on the D6-brane world-volume of the configurations in section 4.2 and using the massive T-duality rules of [17, 24] yields[3]

$$C^{[0]} = \chi + mz \tag{5.1}$$

$$C^{[2]} = -\, C^{[0]} Q * \mathrm{d}h + J^{[2]}\,, \tag{5.2}$$

where χ is the vacuum expectation value and the current $J^{[2]}$ satisfies

$$\mathrm{d}J^{[2]} = 2\pi\ Q\ C^{[0]}\ \delta_p\,. \tag{5.3}$$

The exact forms of $C^{[6]}$ and $C^{[8]}$ depend on the (yet unknown) metric.

Notice in passing that restricting to the branes $i : M^{1,5} \longrightarrow X$ the pull-backs $i^*[G] = 0$ vanish, and any spin$_c$-manifold $M^{1,5}$ satisfies the consistency condition

$$i^*[H] = W + [D] \tag{5.4}$$

for a brane to wrap on it as it is stated in [25]. Here $H = H_{\mathrm{NS}}, H_{\mathrm{RR}}$, $[D]$ is the Poincaré dual of the (here absent) boundary of a three-brane ending on the NS5/D5-brane, and W is some characteristic class constructed naturally out of the topology of the brane. Most notably it could be $\beta(w_2)$, where $\beta : H^2(X, \mathbb{Z}_2) \longrightarrow H^3(X, \mathbb{Z})$ is the Bockstein homomorphism — or possibly the characteristic class of the brane, if it happens to be a gerbe, cf. section 7.

[3]The Hodge star refers again to the metric on S^3 with volume 2π.

5.1 Type IIB charges

The conserved charges are

$$q_7 = \int_{S^1} \mathrm{d}C^{[0]} = ml_B \tag{5.5}$$

$$q_{\mathrm{D5}} = \int_{S^3} \mathrm{d}C^{[0]} \wedge Q\mathrm{d}^\dagger H \tag{5.6}$$

$$q_{\mathrm{NS5}} = \int_{S^3} \mathrm{e}^{2\phi} G - \frac{1}{2}\mathrm{d}\left(C^{[0]}\right)^2 \wedge\ Q\mathrm{d}^\dagger H\,. \tag{5.7}$$

The eight-form charge signals the presence of D7-brane sources cf. section 6.2. If the bounding sphere S^3 here is the same as the one in the IIA picture we obtain the NS5-branes we started from.

Let us show now that it is possible to choose the topology in such a way that a non-zero flux could be captured by some other three-surface than the previously used S^3. We could choose this surface to be a circle bundle $\sigma : M^3 \simeq S^3 \longrightarrow S^2$ with the same base manifold S^2 we used for the cylinder $\pi : C \longrightarrow S^2$. The fiber will be the isometry direction S^1. Above, we got a generator of $H^2(S^2, \mathbb{Z})$ by pushing forward $\mathrm{d}^\dagger(H - H_1)$ by π. This is the Poincaré dual of the fiber of the cylinder bundle, and only well defined in C. Let us now choose a section of the cylinder bundle $i : S^2 \longrightarrow C$. The form $i^*\mathrm{d}^\dagger H$ is again proportional to the generator of $H^2(S^2)$ and carries information about the choice of fibration. We can pull this form back to M^3 using σ. Assuming that $H^1(M^3) = 0$ any closed one-form is exact, and $\sigma^* i^* \mathrm{d}^\dagger H = \mathrm{d}(\omega_h + \omega_v)$ where ω_v is an arbitrary closed vertical form. There is indeed a field that we should associate with the vertical form, namely $\omega_v = \mathrm{d}C^{[0]}$. These choices of the components of $\omega_{h,v}$ carry information about how the isometry direction $S^1 \in z$ is embedded in M^3. The D5-brane charge turns out to be related to the linking number (Hopf invariant)

$$\mathcal{H}(\sigma) = \int_{M^3} \omega \wedge \mathrm{d}\omega = \frac{4\pi}{Q\ \mathrm{vol}(S^2 \subset S^3)} \int_{M^3} \mathrm{d}C^{[0]} \wedge B\,, \tag{5.8}$$

where $\omega = \omega_h + \omega_v$.

The charges obtained from integrating over the two different spheres span a sublattice

$$Q\ \mathbf{q}\ +\ Q\mathcal{H}(\sigma)\ \mathbf{p} \tag{5.9}$$

in the space of NS5/D5-brane charges. The basis here is given in terms of the moduli ϕ and χ as

$$\begin{aligned} \mathbf{q} &= \left(0,\ 2\pi\mathrm{e}^{2\phi}\right) \\ \mathbf{p} &= \frac{1}{4\pi}\ \mathrm{vol}(S^2)\ \left(1,\ \ -2\chi\right), \end{aligned} \tag{5.10}$$

where the ordered pair on the right-hand side refers to the charges in (5.6)–(5.7). Periodicity of $z \in S^1$ makes the modulus χ periodic as well.

5.2 Topology of the ansatz

Let us recapitulate the construction thus far. In type IIA the configuration was described in terms of a gerbe. We were considering a system that included NS5-branes, and locally defined D6-branes that ended in the NS5-branes. If there was a compact isometry direction in the world-volume of the D6-brane we could T-dualize the system. We considered, for simplicity, a dual system with parallel NS5-branes and D5-branes. It turned out that the D6-brane flux could get entangled with the isometry direction in such a way as to produce D5-brane charge in addition to the previously found NS5-brane charge. Hopf invariants classify knots on S^3 so that, in a sense, we have tied the type IIA D6-brane into a knot on the type IIB side.

The obtained geometry, though maybe convoluted if described as above, is not unfamiliar. We may combine the isometry circle and the fiber of the cylinder C into another cylinder C' — the transverse space has then the structure of a cylinder bundle over a sphere, where the cylinder C' is the fiber and the sphere is the usual base space. This space can then be related to an elliptically over a sphere fibered four-manifold with a singular fiber. Furthermore, there is D7-brane charge, which implies a deficit angle in the IIB metric. This charge is associated to a world-volume which is obtained from the D6-brane $\mathcal{M}(D6)$ modding out by isometries S^1, and adding the sphere that has been serving us as a base manifold, or as $\mathcal{M}(D7) = (\mathcal{M}(D6) \times M^3)/S^1$. In section 6.2 we shall use global consistency conditions to patch these local descriptions together and relate the geometry of the system to an F-theory compactification.

6. Generalizations

The construction can be naturally generalized for a finite set of extracted points

$$\mathcal{P} = \{p_\alpha | \alpha \in \mathcal{I}\} \subset S^3 \,, \tag{6.1}$$

by just repeating the above procedure for each point separately. One then obtains for each $\alpha \in \mathcal{I}$ a pair of sets $\mathcal{U}_0^\alpha$ and $\mathcal{U}_1^\alpha$. Let us define $B_0^\alpha = Q\mathrm{d}^\dagger H^\alpha$ and $B_1^\alpha = Q\mathrm{d}^\dagger H_1^\alpha$ in an obvious generalization of section 4.2. We now have to ensure that the arising line bundles on each two-fold intersection obey the condition (3.4).

The open neighborhoods of the fixed points can be chosen so that they do not intersect. We shall have to consider the following intersections of local charts

a) For intersections $\mathcal{U}_1^\alpha \cap \mathcal{U}_0^\alpha$ nothing new happens, and we get the bundles $\mathcal{L}_\alpha(p_\alpha)$. The Chern class was

$$-m(B_0^\alpha - B_1^\alpha) = Qm\ \omega_{p_\alpha} = c_1\Big(\mathcal{L}_\alpha(p_\alpha)\Big) \,, \tag{6.2}$$

where ω_p stands for the (properly normalized) two-form in (4.18).

b) On $\mathcal{U}_0^\alpha \cap \mathcal{U}_0^\beta$ we call the bundle $\mathcal{L}_{\alpha\beta}$. This intersection is again topologically a cylinder (three-sphere minus 2 points), and one finds

$$-m(B_0^\beta - B_0^\alpha) = Qm\ (\omega_{p_\beta} - \omega_{p_\alpha})\,. \tag{6.3}$$

The Chern class is hence the difference of the separate Chern classes

$$c_1\Big(\mathcal{L}_{\alpha\beta}\Big) = c_1\Big(\mathcal{L}_\beta(\beta)\Big) - c_1\Big(\mathcal{L}_\alpha(\alpha)\Big)\,. \tag{6.4}$$

c) The last cases to define are the intersections $\mathcal{U}_0^\beta \cap \mathcal{U}_1^\alpha$. Now we get

$$-m(B_1^\alpha - B_0^\beta) = -Qm\ \omega_{p_\beta} = c_1\Big(\mathcal{L}_\beta(p_\alpha)\Big)\,. \tag{6.5}$$

On the triple intersection $\mathcal{U}_1^\alpha \cap \mathcal{U}_0^\alpha \cap \mathcal{U}_0^\beta$ we observe

$$c_1(\mathcal{L}_\alpha(p_\alpha)) + c_1(\mathcal{L}_{\alpha\beta}) + c_1(\mathcal{L}_\beta(p_\alpha)) = 0\,, \tag{6.6}$$

which means that the product bundle

$$\mathcal{L}_\alpha(p_\alpha) \otimes \mathcal{L}_{\alpha\beta} \otimes \mathcal{L}_\beta(p_\alpha) \tag{6.7}$$

is indeed trivial.

Irrespective of the number of points, the characteristic class is

$$[H] = Q[V] \in H^3(X, \mathbb{Z})\,. \tag{6.8}$$

Notice also that base spaces of the line bundles cannot be described as simple embeddings into space-time. They are entirely properties of the two-fold intersections and hence, in a sense, mutually non-local. The gerbe manages to describe the topology of the space $S^3\backslash\mathcal{P}$; in our case it is merely a catalog of the various non-trivial two-cocycles in $H^2(S^3\backslash\mathcal{P}, \mathbb{Z})$. This geometric picture is similar to that in e.g. refs. [26, 27].

6.1 Consistency and F-theory on a Calabi–Yau three-fold

In addition to verifying the equations of motion, which is a local statement of the stability of the system, we should also consider global consistency. In particular, due to the deficit angle found above it is not consistent to consider only one point $\mathcal{P} = \{p\}$. We should also take care that the system is at rest, which amounts to picking the points in the set $\mathcal{P}$ where the flux comes out of the sphere in a symmetric way. It follows that some discrete subgroup Γ of SO(4) should act as permutations in $\mathcal{P}$. The system will therefore look as if the NS5-branes were put on the top of an orbifold singularity in the origin — the gauge theory on its world-volume is therefore the one described in [28, 29].

Locally in type IIB the transverse space will then look for $\Gamma = \mathbb{Z}_n$ like a Taub–NUT space. In e.g. [30] the B field was set proportional to the self-dual harmonic two form of the Taub–NUT space, the deformations of which contributed to the number of zero modes that were needed in completing the dyon spectrum. This provides a type IIB interpretation for the ubiquitous factor vol(S^2) that appeared in various charges. In our construction it characterized the choice of the base manifold $S^2 \subset S^3$ using which we tie the sphere S^3 together with the compact direction in the Scherk–Schwarz compactification.

6.2 Geometry of the ansatz

In section 5.2 we saw that the configurations that support the dyonic fivebrane charges have an elliptically over a sphere fibered transverse space, and that there is D7-brane charge. Above we found that a stable solution should be invariant under Γ. We have now to patch these local cylinder bundles together in a way that guarantees the consistency of the global metric. All of this is rather suggestive of an orbifold limit of a K3 compactification. Related configurations have been studied e.g. in [13, 31, 32]. A smooth type IIB compactification on a K3 is, however, not the right answer as it would have twice as much supersymmetry as the original NS5/D6-brane system in type IIA. Instead, we shall somehow have to take into account the fivebranes that have been put in the orbifold singularity in order to break the supersymmetry properly to a quarter of the full type IIB supersymmetry.

Let us consider for a moment the type IIB system without the fivebranes. The fact that our system involves an axion that depends on the coordinates of the elliptic fiber was just a consequence of the Scherk–Schwarz reduction. On the other hand, any Scherk–Schwarz compactification of type IIB on a circle can be formulated as an F-theory compactification on a torus bundle over that circle [33]. In particular, an F-theory solution with the same supersymmetry, a monodromy around the D7-branes/orientifolds that shifts the axion, and the same weak coupling orbifold limit of a K3 singularity was already obtained in [34, 35, 36]: The compactification there was an elliptically over $\mathbb{C}P^1 \times \mathbb{C}P^1$ fibered Calabi–Yau three-fold that reduced in ten dimensions in weak coupling to type IIB theory on K3 $\times\, M^{1,5}/(-)^{F_L} \cdot \Omega \cdot \sigma$ in the notation of [36].

Compactification on K3 and orientifolding by $(-)^{F_L} \cdot \Omega \cdot \sigma$ both break supersymmetry by a half, so that in the end we have the right amount of it. The fact that the third cohomology group of a K3 vanishes reflects the fact that we must put the fivebrane dyons in the orbifold singularities of the K3. Also the third cohomology group of $\mathbb{C}P^1 \times \mathbb{C}P^1$ vanishes. We see that the dyon charges should arise from topological winding numbers only on the level of the full Calabi–Yau three-fold whose third cohomology is non-trivial. We conclude that we have found at least a candidate for an F-theory compactification that in type IIB gives rise to the right RR charges and the right NS5-brane charge. We interpret this as an F-theory argument for, even if not a proof of, the consistency of the dilaton and the graviton equations of motion.[4]

[4]Notice, however, that orientifolding imposes restrictions on the two-form potentials.

7. K-theory classification of branes

It has been suggested in [6, 7] and further elaborated in [8, 9, 10] that K-theory of the world-volume of the brane classify D-brane charges. The mapping from the brane into the pertinent K-theory class [7] uses the normal bundle of the embedded submanifold, which should have a spin$_c$-structure. A non-trivial $W \in H^3(Z,\mathbb{Z})$ obstructs such a structure, and it follows from the consistency condition (5.4) that the the pull-back of the NS three-form field strengths vanish.

This situation was generalized to the class of configurations involving a NS field strength that is purely torsion in integral cohomology. In this case one was lead to consider the K-theory of *twisted bundles* [7]: These are bundles whose transition functions do not necessarily satisfy (2.5) but instead

$$(\delta g)_{\alpha\beta\gamma} = h_{\alpha\beta\gamma}\phi_{\alpha\beta\gamma}\,, \tag{7.1}$$

where $\phi_{\alpha\beta\gamma}$ is the obstruction to spin$_c$-structure and $h_{\alpha\beta\gamma}$ is the lift of $[G]$ from $\check{H}^3(X,\mathbb{Z})$ to $\check{H}^2(X,\mathrm{U}(1))$. These objects are trivial gerbes.

7.1 Branes ending on branes

What does happen then, when the restriction of $[G]$ does not vanish on the brane? This leads one to consider branes ending on branes [7]. The world-volume theory will then contain couplings of the form $C - \mathrm{d}L$ of section 2.1 (cf. [13, 14]) and the present considerations are applicable.

The consistent treatment of the world-volume theory involves gerbes along the lines of the construction in section 4.2, and the characteristic class represented by the three-form field strength $[G]$ is, again, that of the gerbe on the brane. In the present construction the identification of the NS three-form field strength as the characteristic class of the gerbe motivates the point of view that the NS three-form incorporates all the topological information of the system — thus including a possible obstruction to spin-structure. This leads us too look upon the consistency condition (5.4) as saying that the left-hand side $i^*[G] + W$, which characterizes the intrinsic topology of the brane whose world-volume theory we are looking at must equal the right-hand side $[D]$, which are the sources. The natural way to write the consistency equation in the form "topology" equals "matter" is then

$$i^*[G] = [D]\,. \tag{7.2}$$

This condition is the direct generalization of the formula of electrodynamics stating that the Chern class of a Maxwell bundle equals the class that characterizes the Dirac string

$$[F] = [2\pi\delta_{\mathrm{string}}]\,. \tag{7.3}$$

Conversely, the obstruction for wrapping a brane on a cycle should be the characteristic class of the gerbe on the cycle.

JHEP07(1999)002

7.2 Classification in type IIA

The type IIA theory presents a couple of problems in the direct application of K-theory: The transverse space has odd dimension, and the analysis in [7] for type IIB has no obvious analogue. Also, the branes to which the cosmological constant m should naively couple are not submanifolds of space-time — after all their *world-volume* should have dimension -1. Nevertheless string theory tells us [16] to treat the cosmological constant $m = R^{[0]}$ on equal footing with any other RR field strength. Finally, on the level of type IIA supergravity the two-form flux is not gauge invariant and it is only defined locally, as we have seen in the previous sections. Instead, the classification group of type IIA D-brane charges has been suggested in [7] to be $K^1(X)$. In [8] this picture was realized by building supersymmetric D-branes starting from non-supersymmetric D9-branes.

The appearance of gerbe solutions calls for an extension of the classification to something that should be the K-theory of gerbes. However, as the complications on the type IIA side are connected with the appearance of the mass parameter, in some cases a straightforward cure is simply to T-dualize the configurations to type IIB theory, where all field strengths couple to geometric objects. This is of course only possible in the configurations that posses at least one compact isometry direction needed to perform T-duality, and will not circumvent the issues that arise in general NS two-form backgrounds. The features that were in IIA understood as consequences of having a cosmological constant then turn into characteristics of a Scherk–Schwarz compactification with respect to a coordinate whose period is related to the type IIA mass parameter. Classification of type IIA D-branes wrapping on a submanifold Z should be done in terms of the pertinent group on the type IIB side, namely $K(Z \times S^1)$, or some modification thereof, where S^1 is the circle with respect to which the duality transformation was performed. Note that the circle may lie also on the D-brane world-volume, in which case one might be compelled to consider $K(Z/S^1)$. As in [7] these considerations seem to point towards the only other available independent K-theory group, namely $K^1(Z)$.

8. Summary

In this article we have investigated some applications of gerbes in string theory and supergravity. We showed how these inherently nonlocal constructions appear naturally in theories that involve massive tensor fields analogously to how fiber bundles appear whenever there is a gauge symmetry. We used this insight in analyzing the NS5/D6-brane systems that appear in massive type IIA supergravity. This leads to quantization conditions that, in particular, relate the cosmological constant to the NS5-brane charge. The D8-branes were interpreted as domain-walls between gerbes. We then proceeded to look for the equivalent T-dual systems on the type IIB side.

JHEP07(1999)002

The dual branes were the NS5-brane and D5-brane dyons. Because the used massive T-duality involved a non-trivial Scherk–Schwarz compactification the axion field became non-trivial, and carried D7-brane charge related to the IIA cosmological constant. The system was identified as an F-theory compactification on an elliptically fibered Calabi–Yau three-fold.

These considerations imply foremost that gerbes are a natural part of string theory. The immediate reason for their appearance on the type IIA side was the zero-form RR field strength, i.e. the cosmological constant. The fact that the corresponding string theory objects would be -1-dimensional renders a direct K-theory classification of D-brane charges difficult on the type IIA side. A straightforward method to find the classification groups for type IIA is then to look at the T-dual objects on the IIB side. We also considered Witten's consistency conditions for brane wrapping and found that they are closely related to gerbe structures. Indeed, the integral cohomology class of the NS three-form should be interpreted as a characteristic class of a gerbe.

We are now in position to use gerbe structures to study nonlocal phenomena, such as branes on orbifold singularities in type IIB. This picture should also prove useful in classifying D-brane charges in the presence of non-trivial NS two-form background. It would also be interesting to see what implications these constructions might have on the spectrum of M-theory.

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